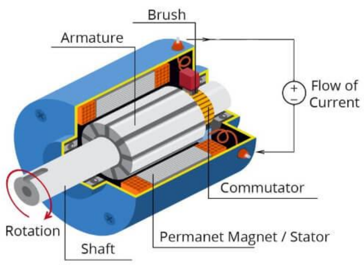
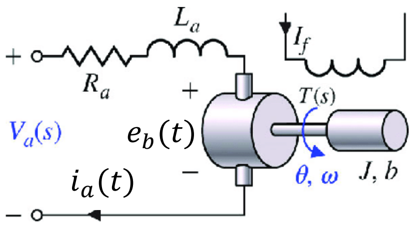
Brief intro to DC Motor

A DC motor consists of three main parts:

1. **Field system** – produces a magnetic field (is constant for permanent magnet system).
2. **Armature** – a rotating coil where current flows and interacts with the magnetic field to produce torque.
3. **Mechanical shaft (rotor and load)** – the mechanical subsystem that converts electromagnetic torque into rotational motion. The shaft dynamics determine how the angular speed evolves in response to the developed torque, friction, inertia, and external load torque.

When a voltage is applied to the armature terminals, current flows through the armature winding.





$e_b(t)$

The armature winding has resistance and inductance. Applying Kirchhoff's Voltage Law around the armature circuit gives:

$$V_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

Here: $V_a(t)$is the applied armature voltage, $R_a$and $L_a$are armature resistance and inductance, $e_b(t)$is the back electromotive force (back-EMF).

When current flows through the armature conductors placed in a magnetic field, each conductor experiences a Lorentz force. The net effect of these forces produces a torque on the rotor. This current interacting with magentic field generates an electromagnetic torque proportional to the armature current.

For a motor operating in its linear region, the electromagnetic torque is directly proportional to armature current:

$$T_m(t) = K_t \, i_a(t)$$

Where $i_a(t)$is the armature current , $K_t$is the torque constant, determined by motor construction and magnetic flux

As the motor rotates, the armature conductors cut magnetic flux and generate a voltage according to Faraday's law. This induced voltage opposes the applied voltage, hence the name back-EMF. This back-EMF is proportional to the motor's angular speed and plays a crucial role in self-regulating the motor.

This back-EMF is directly proportional to the shaft speed**:**

$$e_b(t) = K_b\,\omega(t)$$

At its core, a DC motor is a rotational mechanical system. The motor shaft rotates with angular speed $\omega(t)$, and its motion obeys Newton's law for rotation:

$$J\frac{d\omega(t)}{dt} = (\text{sum of torques acting on the shaft})$$
$$J\frac{d\omega(t)}{dt} = -b\omega(t) + T_m(t) - T_L(t)$$

This equation captures the rotational dynamics of the motor, where J is rotor interia, b is the damping coefficient, T_L is the load torque (incase no load this load torque is zero).

To keep the model simple and suitable for controller design, the following assumptions are commonly made:

- Field flux is constant (field dynamics neglected)
- Magnetic saturation is ignored
- Friction is modeled as viscous friction
- Armature reaction effects are neglected